The damped vibrating system had $\omega_n = 1000$ rad/s and $\zeta = 0.05$. The period of the pulse excitation was 0.023 s and the pulse width was $(t_f/T) = 0.444$. The time domain response and the response spectra are shown in Fig. 4. The response spectra in Fig. 4b contain only the frequencies contained in the excitation function.

Acknowledgments

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Static Analysis of Stiffened Plates

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Nomenclature

A_x, A_y = area of stiffener
a, b = dimensions of plate
D = $Eh^3/12(1-\mu^2)$
E = Young's modulus
h = plate thickness
I = $h^3/12$
I_x, I_y = moment of inertia of the stiffener about the midplane of the plate
q = uniform lateral load
S_x, S_y = moment of area of the stiffener about the midplane of the plate
u, v, w = in-plane and lateral displacements of midplane of the plate

Introduction

In many of the available references on the analysis of stiffened plates, the approximate method proposed by Huber is used. Based on the orthotropic plate theory, this method analyzes the plate stiffener system as a plate of equivalent uniform thickness. It neglects the in-plane displacement of the middle plane of the plate.

In an improved method presented by Clifton et al.,[1] the eccentricity and torsional rigidity of the stiffeners are taken into account, the effect of the stiffeners is smeared out. The governing equations are solved for a simply supported plate and a plate with bridge-type boundary conditions.

In this Note, stiffened clamped rectangular plates have been analyzed taking the eccentricity of the stiffener into consideration. There is no available solution for this problem. The governing equations in terms of the in-plane and lateral displacements of the midplane of the plate are solved using a series consisting of beam functions and Galerkin's procedure. Numerical work has been done for several aspect ratios. The results for the deflection and stresses are presented graphically.

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Differential Equations

The governing differential equations for the eccentrically stiffened plate (Fig. 1) can be written in nondimensional form[1] as

$$a_1 U'' + a_2 V^{\circ\prime} + a_3 U^{\circ\circ} + a_4 W''' = 0 \tag{1}$$

$$b_1 V^{\circ\circ} + b_2 U^{\circ\prime} + b_3 V'' + b_4 W^{\circ\circ\circ} = 0 \tag{2}$$

$$c_1 W'''' + c_2 W''^{\circ\circ} + c_3 W^{\circ\circ\circ\circ} + c_4 U''' + c_5 V^{\circ\circ\circ} = qa^4/Dh \tag{3}$$

where $\bar{x} = x/a$, $\bar{y} = y/b$, $U = ua/h^2$, $V = vb/h^2$, and $W = w/h$. The coefficients are defined in the Appendix. $(\ )'$ and $(\ )^\circ$ denote differentiation with respect to $\bar{x}$ and $\bar{y}$.

The boundary conditions are

$$\text{At } \bar{x} = 0 \text{ and } 1 \qquad W = W' = U = V = 0 \tag{4a}$$

$$\text{At } \bar{y} = 0 \text{ and } 1 \qquad W = W^\circ = U = V = 0 \tag{4b}$$

Solution

The solution that satisfies all of the boundary conditions of Eqs. (4) is assumed to be in the form

$$U = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} x'_m y_n \tag{5a}$$

$$V = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} x_m y_n^\circ \tag{5b}$$

$$W = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} x_m y_n \tag{5c}$$

where x_m and y_n are the beam functions corresponding to the clamped-clamped edge conditions, i.e.,

$$x_m = (\cosh\beta_m\bar{x} - \cos\beta_m\bar{x}) - \alpha_m(\sinh\beta_m\bar{x} - \sin\beta_m\bar{x})$$

$$y_n = (\cosh\beta_n\bar{y} - \cos\beta_n\bar{y}) - \alpha_n(\sinh\beta_n\bar{y} - \sin\beta_n\bar{y})$$

Substituting Eqs. (5) in Eqs. (1-3) and using Galerkin's method, the solution is obtained. The details regarding the values of α_m and β_m and the evaluation of the integrals can be found in Refs. 2 and 3.

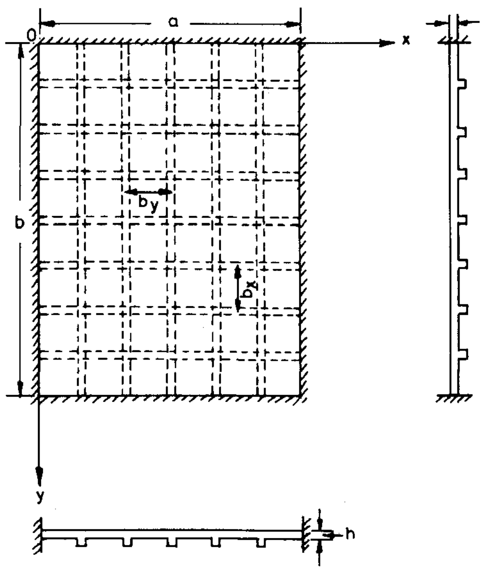

Fig. 1 Plate with eccentric stiffeners.

Table 1 Convergence study and comparison of results for a square plate ($r_{PT} = 0.50$)

No. of terms, $m = n =$	$\bar{W} \times 10^4$	Stress at center $(\bar{\sigma}_T)_y \times 10^2$	Stress at center $(\bar{\sigma}_B)_y \times 10^2$	Stress at edge ($x = a/2$, $y = 0$) $(\bar{\sigma}_T)_y \times 10^2$	Stress at edge ($x = a/2$, $y = 0$) $(\sigma_B)_y \times 10^2$
3	2.1864	−0.2618	0.7820	0.4911	−1.7403
5	2.1998	−0.2867	0.8092	0.5284	−1.7969
7	2.1996	−0.2760	0.7997	0.5253	−1.8223
Ref. 4	2.2000	−0.2816	0.8160	—	—

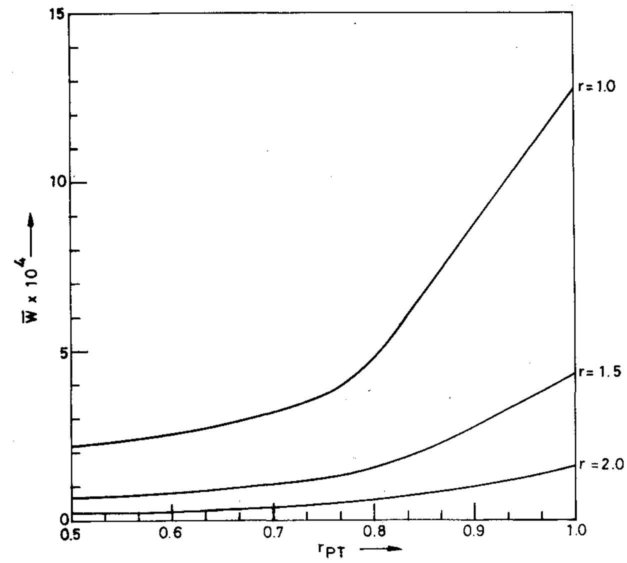

Fig. 2 Variation of deflection vs r_{PT}.

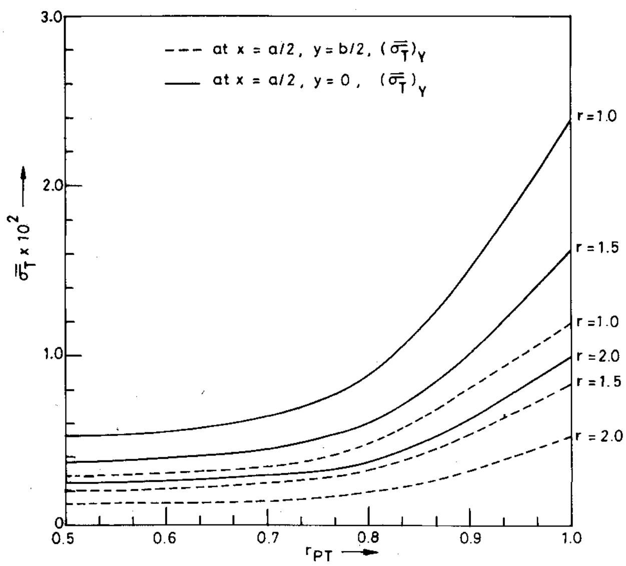

Fig. 3 Variation of stress at top of plate vs r_{PT}.

Numerical Work and Discussion

Rectangular plates clamped at all edges are considered. The cross-sectional area and the spacing of the stiffeners in the x and y directions are kept the same. The ratio of the depth to breadth of the stiffener is taken as four. The spacing between the stiffeners is taken as $16h$, thus

$$r_{PT} = \frac{\text{Volume of the deck plate/unit area}}{\text{Total volume of plate and stiffeners/unit area}} = \frac{h}{\bar{h}}$$

where $h = h + (A_x/b_x) + (A_y/b_y)$.

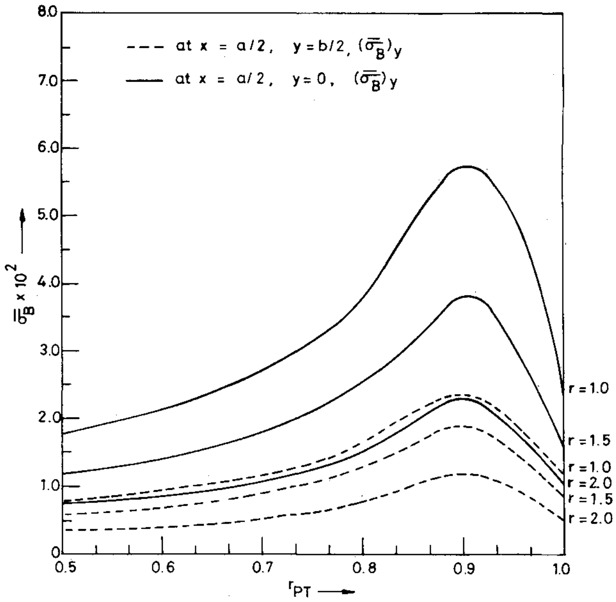

Fig. 4 Variation of stress at bottom of stiffener vs r_{PT}.

The above ratio is varied. The stresses at the top of the plate σ_T and the bottom of the stiffener σ_B are calculated using the expressions

$$\sigma_T = (E/(1-\mu^2))\,[(u' + \mu v) - z_T(w'' + \mu w^{\circ\circ})]$$

$$\sigma_B = E(u' - z_B w'')$$

The deflection and stresses are expressed in the nondimensional form in terms of a reference thickness $\bar{h}$, which corresponds to the thickness of the plate when $r_{PT} = 1$. Let

$$\bar{\sigma}_T = \sigma_T \bar{h}^2/(12qa^2), \qquad \sigma_B = \sigma_B \bar{h}^2/(12qa^2)$$

A convergence study was made by increasing the number of terms in the series, the results of which are given in Table 1. From this table it can be seen that the solution converges well. Thus, it was decided to adopt the five-term ($m = n = 5$) solution for further study. The results for a square plate are compared in Table 1 with those obtained in Ref. 4 using the integral equation technique.

For the parametric study, the r_{PT} ratio was varied from 0.5 to 1.0 and three aspect ratios γ ($= a/b$) of 1.0, 1.5, and 2.0 were considered. The values of the deflection at the center of the plate (Fig. 2) and the stresses (at the top of the plate and the bottom of the stiffener) at the center and edge were calculated. For each aspect ratio, the total volume of material in the plate and the stiffener is constant. Only the r_{PT} ratio is varied. In Fig. 3 the stresses in the y direction at the top of the plate at the center and edge ($y = 0$) are given. Figure 4 shows the stresses in the y direction at the bottom of the stiffener at the center and edge ($y = 0$).

From Figs. 2-4, it is observed that the deflection and stresses decrease as the ratio γ ($= a/b$) is increased. The stress at the bottom of the stiffener increases as the r_{PT} ratio is increased, reaching a maximum value for r_{PT} around 0.9, and then decreases. The controlling value of stress at the center as well as at the edges is the stress at the bottom of the stiffener. It can be concluded (for the cases considered) that, if the magnitude of this controlling stress is to be less than that in a plate without a stiffener, the r_{PT} ratio should be less than 0.65.

Appendix

$$a_1 = 1 + A_x d_x h^2, \qquad a_2 = (1+\mu)\gamma^2/2$$

$$a_3 = (1-\mu)\gamma^2/2, \qquad a_4 = -S_x d_x h$$

$$b_1 = 1 + A_y d_y h^2, \qquad b_2 = (1+\mu)/2\gamma^2$$

$$b_3 = (1-\mu)/2\gamma^2, \qquad b_4 = -S_y d_y h$$

$$c_1 = 1 + 12 d_x I_x, \qquad c_2 = 2\gamma^2$$

$$c_3 = (1 + 12 d_y I_y)\gamma^4, \qquad c_4 = -12 S_x d_x h$$

$$c_5 = -12 S_y d_y h \gamma^4, \qquad \gamma = a/b$$

$$d_x = (1-\mu^2)/(b_x h^3), \qquad d_y = (1-\mu^2)/(b_y h^3)$$

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